Time Microeconomics: Optimization models[1]

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Abstract

This paper provides just a generalization of Becker's theory of allocation of time, allowing joint production, throughout a 2-step optimization process, compatible and avoiding criticism in the sense of Pollak. The model yields some interesting results, analyzed for the static vision.

After doing it, we just expand the vision to a dynamic context, where the stock of time and goods are introduced as explanatory variables. This yields optimal patterns along time for time, goods and commodities, which relates with both the endogenous change of tastes and the habit formation, by including past experiences in the determination of the utility.

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1. **MOTIVATIONS**

“Several notable writers have recently charged that neoclassical economics is ´timeless´. (...) The proper question to ask is not whether neoclassical economics is timeless but whether its treatment of time is adequate. (...) As Hayek recognized many years ago, how time is treated is an important aspect of any explanation of historical change.” (Boland, 1978)

“Economists have always acknowledged the fact that tastes of consumers change (e.g., Marshall in his Principles). But the overwhelming majority took the attitude that it is not their business to be concerned with these changes of taste”(Von Weizsäcker, 1971)

“Actions are constrained(...). Different constraints are decisive for different situations, but the most fundamental constraint is limited time.” (Becker, Nobel lecture 1992)

“while the growing abundance of goods may reduce the value of additional goods, time becomes more valuable as goods become more abundant.”
(Becker, Nobel lecture 1992)

Looking at these citations, it is straightforward to realize about the interest and the power that time, as a decision variable, has in economical aspects, through the way in which it can determine the happiness of the individuals.

As well, it is as easily seen as overwhelmingly ignored by traditional microeconomics (*paraphrasing* Von Weizsäcker) that changes of tastes may vary along time. Marshall was concerned from the very beginning about that:

"(...)and the man is not the same at the beginning as at the end of it”
(A. Marshall, “*Principles of Economics,”*, 1962.)

The behaviour of the individuals, the way it changes along time, and overall the manner in which both make impact on the welfare of individuals and on the society itself is a very challenging aspect, usually and mostly taken as given throughout a non-variable utility function.

But, as shown above, there exist some literature by some of the main references in the economical thought, and not by many others. Even in the paper *Dynamic Utility*, by Ragnar Frisch (Econometrica © 1964), he worried about it.

Once showed the importance and interest of this topic, it must be shown, as well, the importance of the nature of the individuals:

“*Divitae bonum non sunt* (Material wealth is not the one good)”, “*Rationale est homo* (A man is rational)” and “*Res severa est verum gaudium* (Happiness is a serious thing)”, in *De Beneficiis*, by Lucio Seneca, spanish philosopher and Nero’s counsellor during Roman Empire .

2. CONEXION WITH RELEVANT LITERATURE

It could be good to start talking about the historical line that research in time issues within the field of economics has followed.

The problem is that, on this way, one always finds different approaches, and not all of them have been developed very much, for several reasons, although several top researchers in economics along the time have met or crashed this topic at some point of time in their research. It is the case of, for example, Becker, Hayek, Samuelson, Marshall, Hicks, Koopmans or Debreu, for citing some ones.

The diversity of point of views, and even of purposes, has made this research particularly heterogeneous, perhaps because none of them wanted explicitly to research either the time use or the change along time itself, but indeed to finish in other conclusions for which the presence of time use in their models was crucial, in one way or another. The benchmarks on these theories of allocation of time are Becker and Gronau contributions, though, following Boland, in Becker's "*There is no reason for historical change; hence it cannot be explained"* (Boland, 1978)

One paper in the existing literature making a good compendium of what has been the research in time issues is *Time in Economics vs Economics in Time: The 'Hayek Problem'*, by L. A. Boland published in the Canadian Journal of Economics / Revue canadienne d'Economique © 1978 Canadian Economics Association.

We conceive time in both the time use perspective, and the time line one. Our implicit point always is the idea of time uses depending on time (the time line, i.e, time use varying along time). That is why Boland´s paper is of interest, since it is somehow connecting both perspectives, analyzing how they have been evolving along time in the literature.

In Boland ´s paper, a great analysis is made, whose summarize about the different approaches in using time in economics can be read as follows, looking at time as both the time line (decisions *along* time) and the time use (decisions *about how to use* the time):

- ✓ **Time and static models**: even in a standard model in economics, implicitly is the idea of time, whenever, for example, we make comparative static. The problem is, as Boland argues, this always is a matter of interpretation of the model, and *"Therefore, with respect to any given model, today´ s values of the endogenous variables may be shown to be consistent with today´ s values of the exogenous variables, but tomorrow their respective values may not be consistent. Since dynamic processes obviously refer to more than one point in time, the explanatory usefulness of a static model would seem rather limited"* (Boland´ s, pp.2)
- ✓ **Time-based variables**: during the late 50s, Koopmans (1957) and Debreu (1959) introduced time-based variables, i.e., subscripts making reference to the point in time in which the goods are consumed, suggesting in this way that, as Boland comments, a hamburger is not the same hamburger for the consumer at time t´ than at time t´´. Although very interesting, and quite used, this approach receives a criticism in

Boland´ s, since he argues there is no dynamics in this model, since formally the model is like a static model over all the time range, which only multiplies the number of goods. Nevertheless, this idea is commonly used in economics.

- ✓ **Time preferences or the economics of time:** This approach deals with the idea of including time as a commodity. Bohm-Bawerk (1889) and Becker (1971) are examples of it, and the impact of the Becker´ s contribution is something to highlight. The former focuses more in production theory, while the latter develops more the consumer side, relating it to several fields. But, again, there is no dynamics in here. Time is something exogenous and static variable. *"Neither Becker´ s nor Bohm-Bawerk´s can avoid the static approach of the givens (the constrains, the tastes, the production functions, time available, etc) […] There is no reason for historical change; hence it cannot be explained"* (Boland´ s pp.5)
- ✓ **Variable givens or lagged variables:** This is an alternative approach, attempting to determine the time path trajectory of the endogenous variables, and that change is suggested to be done because of a change in the parameters or in the exogenous variables along time, or both, being Hicks´ model (1971) an example for the former, where he talk about an "autonomous invention", and Kaldor´ s growth model another one for the latter. Of course we are not forced to assume that the period of change for the exogenous variables has to be the same for the endogenous, and an example of this is Von Neumann balanced growth model. It is worthy to be read this section, specially the part in which he establishes a parallelism about the dealing what in economics is done to a point in time and what is (in physics) the dealing of a point in space.
- ✓ **Flow variables:** Similarly, this approach is one such that is extending a static vision of a model to a dynamic one by inserting appropriately differential equations, and examples in the literature can be found in Barro and Grossman (1971) or Arrow (1959).
- ✓ **Time, logic and true statements:** this part is related to the discussion about the neoclassical economics to be or not to be timeless, as many authors suggested in the 70s, since some of them tells that *"all economical analysis has been merely logical derivation of solutions"* as Boland writes in reference to Georgescu-Roegen (1971) and Shackle (1972) contributions. This is a controversial point in Boland´ s, very discussed, so we do not enter in details here.
- ✓ **Time and knowledge: the Hayek problem:** at this section, Boland insist more in the idea that in previous sections, the way in which time is included suggest that any reliance on only standard general equilibrium theory precludes and discard an explanation for historical changes, since all the causes, motivations and reason for changes are beyond explanation since they are exogenous to the models. He points as well that Hayek (1937) realized about this problem, and that this remains an essential consideration in most Austrian models, as in Hicks (1973) and in Lachmann (1976). This is what Boland understands as the "Hayek problem", and the same Hayek in 1937 showed and recognized in his paper his incapability to solve it, but pointing in such very abstract paper a lot of insights, one of them collected by Boland, related to knowledge,

what in turn relates with the next part we are going to relate with a little: habit formation. Boland, to conclude, suggests that the individual process of acquiring knowledge must be *endogenous*, and that the individual decision and process of learning/adapting must be taken *in real time*.

As we can observe in Boland´ s, his work is a good and quite complete description, introduction, motivation and connexion to any existing research related to how to include time in economic models, and the implication it implies, in different fields.

Other fields this project is having touch and/or impact with, directly or indirectly, are such that habit formation, endogenous tastes or dynamic utility.

Related to *habit formation* and *endogenous tastes*, there exist some very interesting work by Pollak(1968, 1970, 1973, 1976), the so called addiction perspective by Becker along his career, Hammond (1975), Weizsäcker (1970), Gorman (1967), Peston (1967) or Samuelson (1956), where none of them uses time use as an explanatory variable, for this concrete purpose.

According to *dynamic utility* we can find literature in Frisch (1964).

All of them, mostly, fit some of the Boland's specified categories of how time, -in the broad sense, as time *use* and time *line*-, has been modelled in economics, and which we have tried to summarize above.

Our project is comprising, within the Boland´ s points summarized above, almost all points almost straightforwardly but the last two at the same time (this itself could be an interesting contribution), and in reference to the last one, the Von Hayek problem, we deem is on the line of the proposed solution Boland suggests, since the process is endogenous and in real time. However, all this discussion (which comprises as well the second last point in our Boland´ s summarize) is mainly based on Von Hayek´ s reflections and thoughts in what he recognizes himself is a very ambiguous paper (and quite abstract), which state what later on Boland calls the "Von Hayek problem". Due to this ambiguity on the edge of at least Von Hayek´ s capabilities, as he recognized, our value over our own job might be incorrect, but still might be a possible solution or at least a small contribution to the solution to the Von Hayek's problem applied to consumer theory with time inputs.

3. INTRODUCTION

This *serious* task of happiness for individuals, then, should start with a very long philosophical speech (and nowadays in modern societies Séneca's philosophy applies very well), since, in the end, we think that any issue related with behaviour, decision and happiness of human beings is something that escapes surely any present knowledge in a formal scientific way.

As we do not consider ourselves experts in that task, we simply claim to one point:

What is, actually, the only thing we can, -as human beings, as individuals, or as consumers-, choose?

Probably, this statement, -as simple as exigent-, if is thought carefully and with *time* enough, becomes to have a much easier response we thought at the beginning.

Our point of view is simple: we think that, in the end, we can only choose one thing:

***How to distribute the time that has been given to us, for the mere fact of existing and being alive*.**

This is simply our starting point and only main assumption. However, this might be a very stoicism perspective. Human beings do enjoy material things too, in a very hedonic way, and it must not be ignored. Up to Becker (1965) time was not formally introduced into an economic model, though previous works by Mincer pointed the importance of doing it. Hence, we ended with a model in economics that is modelling both hedonism and stoicism, relating it to economic concepts. Along this paper we just try to generalize Becker's contribution, and putting it into a dynamic context.

Then, we build a generalized static model in a 2-step optimization process, keeping the essence in Becker, with the time use and market goods as inputs for the attitude that individuals demonstrate as producers of commodities, firstly, and, with commodities as the source of utility or happiness for them, secondly.

Later on, we just make the dynamic version of the provided Generalized Becker Theory of Allocation of Time, introducing for the first time the stock of time used as an explanatory variable. By solving it, we get optimal patterns for demands of time and goods, along time, and their stock accumulated along time, which in the case of the time use, points to the habit formation, understanding habit as in the dictionary is defined: *a pattern of behaviour acquired through frequent repetition.* And for each period of time, then, ´optimal schedules´ are defined, let us say, which is realistic, since we all do make plans.

4. STATIC MODEL

The main idea turns around including as the main argument having impact on the utility function the concept of 'commodities', something for which you have to use inputs in order for it to be produced. These inputs are market goods and time spent in the activities related to the commodity[3].

Hence, the problem includes time as an input, and requires and states a time constrain in addition to the usual budget constrain.

The Becker's model looks like this:

$$\max_{x,T} \; U\left(\vec{Z}\right) = U\left(f_1\left(\vec{x}_1, \vec{T}_1\right), \ldots, f_m\left(\vec{x}_m, \vec{T}_m\right)\right)$$

$$s.t. \quad \sum_i^m \vec{p}_i^T \vec{x}_i \leq I = \vec{w}^T \vec{T}_w + V$$

$$\sum_i^m \vec{T}_i \leq \vec{T} - \vec{T}_w$$

where:

- $Z_i = f_i\left(\vec{x}_i, \vec{T}_i\right)$, with $Z_i \in \mathbb{R}^m, i = 1, \ldots, m$ denotes the commodity *i*, for which achievement the vectors $\vec{x}$ and $\vec{T}$ are needed.
- $\vec{x}_i$: vectors of demands of goods used to produce commodity *i*, whose associate price vector is $\vec{p}$.
- $\vec{T}_i$: vectors of time inputs used to produce commodity *i*.
- $\vec{T}$: vector of total time available for each type of time (whose elements must add up to 24 hours a day, 7 days a week, etc)
- $\vec{T}_w$: vector of time devoted to the job in the job market, whose wage is *w* units of money per unit of time.
- *V*: other income

By obtaining in the second constrain in the problem an expression for $\vec{T}_w$, and substituting it in the first constraint, Becker comes to the following constrain:

$$\sum_i^m \vec{p}_i^T \vec{x}_i + \sum_i^m \vec{w}^T \vec{T}_i \leq \vec{w}^T \vec{T} + V = S$$

the ***<u>Full Income constrain</u>***.

[3] In Becker (1965) the model is presented as we try to sketch in here. Nevertheless, it should be pointed out that later on, in the book *Economic Theory* by Becker a new input is introduced. Such input is the set of environmental variables, as he names them. By simplicity, and given that it is not related to the allocation of time directly, we do not include this in our sketch of the model, though we let it for future research.

Now we proceed suggesting a more general framework than in Becker's huge contribution, which has been very well accepted in Economics, finishing our suggestion showing how Becker's theory of allocation of time is a particularization of it.

First, let us expand Becker's model to allow it to present joint production. The model below allows joint production, as can be seen here, sequentially:

What Becker does, concerning to the production of commodities, is the following:

$$Z_i = f_i\left(\begin{pmatrix} x_{1i} \\ \\ x_{ni} \end{pmatrix}_i, \begin{pmatrix} T_{1i} \\ \\ T_{pi} \end{pmatrix}_i\right), \text{with } Z_i \in \mathbb{R}^m, i = 1, \ldots, m$$

Which does not allow to joint production, what is actually daily life[4]. In Becker's framework, if you use some time to produce one commodity, such a time used is not allowed to produce any other commodity at the same period of time. An example showing Becker's limitations is that something so common as to "produce meals" cooking and "produce listening to music", which are two commodities, cannot be done at the same time. So, if you are cooking, you just cook, to get the meal, nothing else; if you want to listen music, to enjoy the listening to music, you just stay listening (maybe sat down? Imagine if you are just dancing, studying or in the shower, for instance).

To introduce joint production, you just have to expand each production function for each commodity in Becker's to matrices, as follows:

[4] For the sake of this very important issue, it must not be forgotten Pollak´s comment: "*The major problem in studying the allocation of time in the household production function model centers on joint production rather than nonconstant returns to scale.*" (Pollak, 1975)

$$Z_i = f_i\left(\begin{pmatrix} x_{11} & x_{1q} \\ x_{n1} & x_{nq} \end{pmatrix}_{n\times q}, \begin{pmatrix} T_{11} & T_{1r} \\ T_{p1} & T_{pr} \end{pmatrix}_{p\times r}\right)$$

with $\vec{Z} \in \mathbb{R}^m, i = 1,\dots, m,$ as commodities,

where $X_{n\times q} = \begin{pmatrix} x_{11} & x_{1q} \\ x_{n1} & x_{nq} \end{pmatrix}$ is the matrix of types of goods (by lines) used in each different use (by rows) one can use them,

and $\Im_{p\times r} = \begin{pmatrix} T_{11} & T_{1r} \\ T_{p1} & T_{pr} \end{pmatrix}_{p\times r}$ is the matrix of types of time inputs (by lines) used in each different use (by rows) one can use them, where:

$\vec{T}_w = \begin{pmatrix} T_{1w} \\ T_{pw} \end{pmatrix}$ is the time devoted to work, for each different type of time, which is determined as a "residual".

Then, the expanded or ***Generalized (Becker's) Theory of Allocation of Time*** can be represented as follows:

$$\max_{X,\Im} \; U\left(\vec{Z}\right) = U\left(f_1\left(X_{n\times q}, \Im_{p\times r}\right),\dots, f_m\left(X_{n\times q}, \Im_{p\times r}\right)\right)$$

$$s.t. \quad \sum_{q=1}^{q} \vec{p}^{\,T}\vec{x}_q \le I = \vec{w}^T\vec{T}_w + V$$

$$\sum_{r=1}^{r} \vec{T}_r \le \vec{T} - \vec{T}_w$$

where:

$\vec{T}_r$ vector corresponding to the r-th row in $\Im_{p\times r}$ but the working time $\vec{T}_w$

$\vec{x}_q$ vector corresponding to the q-th row in $X_{n\times q}$

Which allows such realistic examples as cooking and listening music to be commodities (or activities, if we prefer to not to follow Becker's terminology and use a more naïve one) produced at the same instant of time, during/with the same time period/input.

It should be noticed that Becker's theory of allocation of time is a particular case of the problem stated above, when $q=r=m$ and the production of the m-th commodity is *exclusively* depending on the m-th row of both $X_{n\times q}, \Im_{p\times r}$.

Not happy with this, we want to walk towards a more realistic approach. As people is, -within Becker framework (and hence, within the generalized version presented before)-, considered to be both producers and consumers, then let us state the following 2-step optimization model, where, in sum, when the individuals are considered as producers they have a cost minimization attitude, and when they are considered as consumers who enjoy consumption, they are utility maximizers.

Let us, then, consider the following 2-step optimization problem:

First step:

$$\min_{X_{n\times q}, \Im_{p\times r}} E\left(X_{n\times q}, \Im_{p\times r}\right) = \sum_{q=1}^{q} \vec{p}^T \vec{x}_q + \sum_{r=1}^{r} \vec{w}^T \vec{T}_r$$

$$s.t.\ \ f_i\left(X_{n\times q}, \Im_{p\times r}\right) \geq Z_i, i = 1 \ldots m \qquad \text{(P1)}$$

which yields:

$$X^*_{n\times q} = \eta\left(\vec{p}, \vec{w}, \vec{Z}\right) \qquad (1)$$

$$\Im^*_{p\times r} = \phi\left(\vec{p}, \vec{w}, \vec{Z}\right)$$

and the corresponding Value Function:

$$E\left(X^*_{n\times q}, \Im^*_{p\times r}\right) = E^*\left(\vec{p}, \vec{w}, \vec{Z}\right) \qquad (2)$$

Where $\vec{Z}$ is the amount of commodities, to be determined in the second step.

Second step:

$$\max_{\vec{Z}}\ U\left(\vec{Z}\right)$$

$$s.t.\ \ E^*\left(\vec{p}, \vec{w}, \vec{Z}\right) \leq \vec{w}^T \vec{T} + V = S$$

where we, recalling Becker, denote as well as ***Generalized Full Income Constraint***, and we get optimal solutions $\vec{Z}^* = \upsilon\left(\vec{p}, \vec{w}, \vec{T}, V\right)$ and hence, as in the first step, the correspondent value function for the problem

$H = h\left(\vec{p}, \vec{w}, \vec{T}, V\right)$, which we denote with *h* of *h*appiness, and for simplicity can be written as well as $H = h\left(\vec{p}, \vec{w}, S\right)$.

It must be noticed that $\vec{T}$ is a parameter which might vary, since it is the total amount of available time to each type of time (which can have different uses). That is not necessarily constant, since might be redistributions within the whole sum of elements in $\vec{T}$. What is immutable is the sum of the elements of $\vec{T}$, which is, indeed, the whole amount of time what is available, i.e., 24h per day, 7 days a week, etc, depending on the length of the time period considered. To clarify, let us state an illustration. Let us define three different types of time: time in the morning, in the evening and at night. Within each one of them, we can perform different uses of each of the types. But what one people can consider morning, evening and night, can be a different amount of time than what other can state. As well, the same individual might consider in different instants of time the morning to be shorter than in others, or the night to be longer than in others. This might be even more different across different cultures or regions all around the world. But in all cases, the only immutable thing is that the sum of the total time available for any type of time is the same, since we all have the same time per period, i.e., 24h a day, 7 days a week and so on.

The value function for the first step is similar to an expenditure function in classical microeconomic theory, and the value function for the second step is similar too to the indirect utility function in classical microeconomic theory.

For a general EMP in classical microeconomics, we recall the expenditure function properties in books like Mas-Colell et al (1995)[5] or Segura (1988)[6] here, adapting them to our case in the first step:

- (Properties of the Expenditure function) Suppose that f_i are continuos household production functions, satisfying the local nonsatiation assumption over the set $\left(X_{n\times q}, \Im_{p\times r}\right) \in \mathbb{R}^{\max\{n,p\}}$. For $\vec{p} >> 0, \vec{w} >> 0$. The expenditure function $E^*\left(\vec{p}, \vec{w}, \vec{Z}\right)$ is:
    - Homogeneous of degree 1 in *(p,w).*
    - Strictly increasing in *Z* and nondecreasing in any element of *(p,w).*
    - Concave in *(p,w).*
    - Continuous in *p* and *Z.*

For the sake of the *happiness function*[7] the classical properties do not apply in general. We just reproduce them again from Mas-Collell et al (1995) or Segura (1988) to comment them later:

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[5] See proposition 3.E.2 in Mas-Collell et. al, *Microeconomic Theory*, Oxford University Press 1995. as the problem is parallel to the one considered by them, the properties we highlight are just an adaptation of what it is written by them.

[6] His book provides more formal explanations in general and in particular to our case.

- Proposition 3.D.3 in Mas-Collell et al (1995) adapted to our case[8]. Suppose that U is a continuous utility function representing a locally non satiated preference relation defined on the consumption set. The indirect utility function $H = h\left(\vec{p}, \vec{w}, S\right)$ is:

   a) Homogeneous of degree 0.
   b) Strictly increasing in $S = \vec{w}^T \vec{T} + V$, the generalized full income, and nonincreasing in any element of *(p,w)*.
   c) Quasiconvex in *(p,w).*
   d) Continuous in *(p,w,S)*

In our case for the second step of the 2-step optimization problem suggested, the only ones holding seem to be properties a) and d). We deem b) and c) not to be so obviously satisfied, given that *w* in our problem is both a price for time inputs and one of the two sources of full income. So what in Mas-Colell or Segura is considered as prices *p*, partly is included in *S,* so it might be so precipitated conclude the last sentence in b). These properties seem to be really interesting, and we hope we can work on them in future research, hopefully as part of a doctoral dissertation.

Now we just focus in the happiness function obtained as a result of the second step in order to show a very interesting fact, result and implication of our suggested model. By total differentiation of the happiness function, we get the following:

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[7] As we denote the whole value function as *happiness*, we prefer to be coherent with the terminology and vocabulary, and forget about the indirect utility, to reduce it to one mere word, instead of two.

[8] Recall that the problem they consider is the classical UMP, not the one we propose, but the notation related to prices and wage holds partly and can be used in order to show the comments made bellow, but realizing of w , wealth, in Mas-Colell being S , full income, in Becker and our generalized version.

$$H = h\left(\vec{p},\vec{w},\vec{T},V\right) \Rightarrow \text{(total differentiation)}$$

$$\sum_{n=1}^{n}\frac{\partial h}{\partial p_n}dp_n + \sum_{p=1}^{p}\frac{\partial h}{\partial w_p}dw_p + \frac{\partial h}{\partial V}dV + \sum_{p=1}^{p}\frac{\partial h}{\partial T_p}dT_p = 0 \Rightarrow$$

(for $dp_n = dp$ the same $\forall n, dw_p = dw$ the same $\forall n$)

$$dp\sum_{n=1}^{n}\frac{\partial h}{\partial p_n} + dw\sum_{p=1}^{p}\frac{\partial h}{\partial w_p} + \frac{\partial h}{\partial V}dV + \sum_{p=1}^{p}\frac{\partial h}{\partial T_p}dT_p = 0$$

$$dw\sum_{p=1}^{p}\frac{\partial h}{\partial w_p} = -\left(dp\sum_{n=1}^{n}\frac{\partial h}{\partial p_n} + \frac{\partial h}{\partial V}dV + \sum_{p=1}^{p}\frac{\partial h}{\partial T_p}dT_p\right) \Rightarrow \text{(dividing by } dp\text{)}$$

$$\frac{dw}{dp}\sum_{p=1}^{p}\frac{\partial h}{\partial w_p} = -\left(\sum_{n=1}^{n}\frac{\partial h}{\partial p_n} + \frac{\partial h}{\partial V}\frac{dV}{dp} + \sum_{p=1}^{p}\frac{\partial h}{\partial T_p}\frac{dT_p}{dp}\right) \Rightarrow$$

$$\frac{dw}{dp} = -\frac{\left(\sum_{n=1}^{n}\frac{\partial h}{\partial p_n} + \frac{\partial h}{\partial V}\frac{dV}{dp} + \sum_{p=1}^{p}\frac{\partial h}{\partial T_p}\frac{dT_p}{dp}\right)}{\sum_{p=1}^{p}\frac{\partial h}{\partial w_p}}$$

$$\frac{dw}{dp} = -\frac{\left(\sum_{n=1}^{n}\frac{\partial h}{\partial p_n} + \frac{\partial h}{\partial V}\frac{dV}{dp} + \sum_{p=1}^{p}\frac{\partial h}{\partial T_p}\frac{dT_p}{dp}\right)}{\sum_{p=1}^{p}\frac{\partial h}{\partial w_p}} \qquad (4)$$

Recalling that:

$$\frac{\partial h}{\partial V} = \mu$$

$$\frac{\partial h}{\partial T_p} = w_p$$

$$\frac{\partial h}{\partial\left(\vec{w}^T\vec{T} + V\right)} = \mu$$

$$\frac{\partial\left(\vec{w}^T\vec{T} + V\right)}{\partial w_p} = T_p$$

And hence, by the chain rule:

$$\frac{\partial h}{\partial \vec{w}^T} = \sum_{p=1}^{p}\frac{\partial h}{\partial\left(\vec{w}^T\vec{T} + V\right)}\frac{\partial\left(\vec{w}^T\vec{T} + V\right)}{\partial w_p} = \sum_{p=1}^{p}\mu T_p = \mu\sum_{p=1}^{p}T_p = \mu T$$

where the scalar T is the total amount of time available (24h per day, etc)

Expression (4) can then be written as follows:

$$\frac{dw}{dp} = -\frac{\left(\sum_{n=1}^{n} \frac{\partial h}{\partial p_n} + \mu \frac{dV}{dp} + \sum_{p=1}^{p} w_p \frac{dT_p}{dp}\right)}{\mu T} \qquad (4^*)$$

If we assume $\frac{dV}{dp} = 0$ and $\frac{dT_p}{dp} = 0, \forall p$, the role of changes in other income (V) is deleted; so it is the price effect on variation of total time available for each type of time[9] (remember, evening, morning, etc) , and we get an expression telling as what should be the wage rate compensation after the change in prices, -which, as in reality, is reduced to one number (the so called CPI)-, in order to keep the same level of happiness for the individuals/workers:

$$\frac{dw}{dp} = -\frac{\sum_{n=1}^{n} \frac{\partial h}{\partial p_n}}{\mu T} \qquad (4^{**})$$

Naturally, for normal goods, then this relation between wages and prices is positive, since $\frac{\partial h}{\partial p_n} < 0, \forall n$, and we see that the higher the valuation of the *full income*, -measured by the shadow price μ, which includes the valuation of both other income and overall the full time individuals have available-, the lower the required wage increase in order to keep individuals as happy as before the wage change.

So then, after analyzing a bit the model suggested, we can say the following:

PROPOSITION 1 – *(Law of wage compensation)*: The higher the valuation of both the whole time and nonworking earned money, the lower the wage increase in order to keep the same level of happiness for the individuals.

Let us now go back, again, to the happiness function:

$$H = h\left(\vec{p}, \vec{w}, \vec{T}, V\right) = U\left(\vec{Z}^*\left(\vec{p}, \vec{w}, \vec{T}, V\right)\right) - \lambda^*\left(E^*\left(\vec{p}, \vec{w}, \vec{Z}^*\left(\vec{p}, \vec{w}, \vec{T}, V\right)\right) - \vec{w}^T \vec{T} - V\right)$$

By considering a marginal wage increase, *ceteris paribus*, as the happiness function is a value function, -and then is maximal-, its partial derivative with respect to the wage vector must be zero.

[9] This term is not deleted by assumption, but indeed by construction, if and only if we assume the same wage *w* for all types *p* of time you can imagine (remember, for example, the case of 3 types of time, in the morning, evening and night, as covering the whole time in the day). In such a case, with same wage rate *w* for all types, then we can extract w as a common factor and as the total amount available is fixed and given (24h per day, etc), then, the sum of all variations over all possible types of time adds up to zero, whatever the cause. In this case is the cause is price changes. We should be careful with this constant wage assumption, since it is commonly known that wages are not the same, for example, when working at nights.

$$\sum_{p=1}^{p}\frac{\partial h\left(\vec{p},\vec{w},\vec{T},V\right)}{\partial w_p}=0=\sum_{i=1}^{m}\sum_{p=1}^{p}U'_{Z_i^*}\frac{\partial Z_i^*}{\partial w_p}-\lambda^*\left(\sum_{p=1}^{p}\frac{\partial E^*}{\partial w_p}+\sum_{i=1}^{m}\sum_{p=1}^{p}\frac{\partial E^*}{\partial Z_i^*}\frac{\partial Z_i^*}{\partial w_p}-T\right)$$

Let us assume first that no change in inputs of goods is held.

If $\sum_{p=1}^{p}\frac{\partial E^*}{\partial w_p}+\sum_{i=1}^{m}\sum_{p=1}^{p}\frac{\partial E^*}{\partial Z_i^*}\frac{\partial Z_i^*}{\partial w_p}-T>0,$ as $\lambda^*>0$ since it is the lagrange multiplier then it must occur $\frac{\partial Z_i^*}{\partial w_p}>0$ for some i and some $p,$ since by assumption, $U'_{Z_i^*}>0,\forall i$ in order to satisfy the initial partial derivative of h to be zero.

The fact $\frac{\partial Z_i^*}{\partial w_p}>0$ implies that at least for one commodity i and one wage associated to a type of time p, it must exist some $T_{pr}\in\Im_{p\times r+p}:\frac{\partial T_{pr}}{\partial w_p}>0$, which means that such specific time input T_{pr} (some time input of type p used in a manner r) is a Giffen good, if we assume the production of commodities to be at least nondecreasing for all time inputs, which is reasonable.

What it is not obvious is under which conditions $\sum_{p=1}^{p}\frac{\partial E^*}{\partial w_p}+\sum_{i=1}^{m}\sum_{p=1}^{p}\frac{\partial E^*}{\partial Z_i^*}\frac{\partial Z_i^*}{\partial w_p}-T>0$ is holding (if it is possible it to hold), and we let it for future research, again, since exceeds the purpose for this paper, though may be an interesting point within a doctoral dissertation. Our intuition is that it may be a household technology such that our premise $\sum_{p=1}^{p}\frac{\partial E^*}{\partial w_p}+\sum_{i=1}^{m}\sum_{p=1}^{p}\frac{\partial E^*}{\partial Z_i^*}\frac{\partial Z_i^*}{\partial w_p}-T>0$ holds.[10]

PROPOSITION 2 – ***(Some type and use of leisure time is a Giffen good)***: Under certain conditions over the expenditure function (*E)* and the household technology (*Z)* functions, it must be the case that some type of leisure time used in a certain manner is a Giffen good.

[10] It might be useful to wonder ourselves about the plausibility of the linear cost function assumed since the beginning of our 2-step model, for this purpose, and not only rely on the household technology for the sake of $\sum_{p=1}^{p}\frac{\partial E^*}{\partial w_p}+\sum_{i=1}^{m}\sum_{p=1}^{p}\frac{\partial E^*}{\partial Z_i^*}\frac{\partial Z_i^*}{\partial w_p}-T>0$ to be fulfilled, and hence, consider new functional forms for *E*.

After discussing our suggested model, then it is easily seen how Becker's model written at the beginning of this section is just a particular case of our model. In his paper, Becker continues on the way of getting closer to the traditional microeconomic model, and then, comes a quite crucial and discussable assumption, where the demands of goods and time inputs for each activity is supposed to be a fixed proportion of the amount of commodities as follows:

$$\vec{T}_i = \vec{t}_i Z_i$$
$$\vec{x}_i = \vec{b}_i Z_i$$

Which inserted into the full income constraint, yields:

$$\sum_i \vec{p}_i^T \vec{b}_i Z_i + \sum_i \vec{w}_i^T \vec{t}_i Z_i \le \vec{w}_i^T \vec{T} + V = S \Rightarrow \sum_i \left(\vec{p}_i^T \vec{b}_i + \vec{w}_i^T \vec{t}_i \right) Z_i \le \vec{w}_i^T \vec{T} + V = S \Rightarrow$$
$$\Rightarrow \sum_i \pi_i Z_i \le \vec{w}_i^T \vec{T} + V = S$$

The model can be then expressed in this way[11]:

$$\max_{\vec{Z}} \; U\left(\vec{Z}\right)$$
$$s.t. \quad \sum_i \pi_i Z_i \le S$$

Where the prices π_i are expressing both the cost of goods and cost of time, and S is what Becker calls the full income.

The model brings the standard condition of Marginal Rate of Substitution between each pair of commodities equal to the price ratio.

$$\frac{U'_{Z_i}}{U'_{Z_j}} = \frac{\pi_i}{\pi_j}$$

[11] Becker provides as well an alternative formulation in terms of what he calls the loss function, L(Z)=S-I, what is measuring the opportunity cost of time spent doing all the activities in the range, which is measured at the constant wage rate w. the alternative formulation can be expressed as follows:

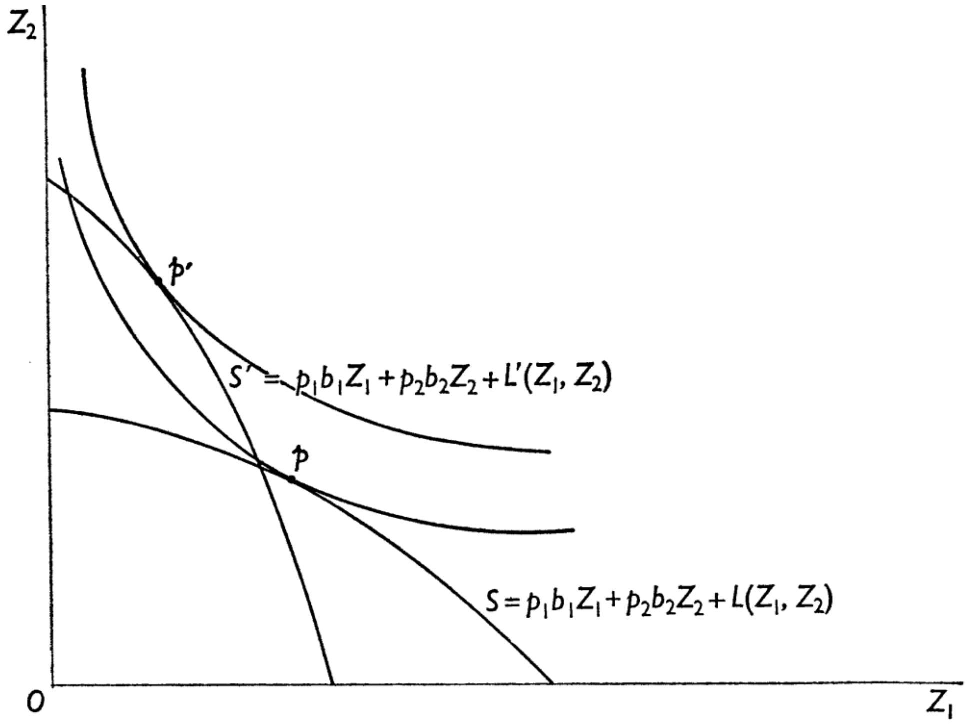
$$\max_{\vec{Z}} \; U\left(\vec{Z}\right)$$
$$s.t. \quad \sum_i^m p_i b_i Z_i + L(Z) \le S$$

with $L(Z) = \sum_i w t_i Z_i$

To get this version we remark the crucial role of being assumed that there is some vector indicating which are the good and time intensities for each commodity. This is a bit controversial and it is discussed later.

At this point, Becker gets a model that looks like the textbook model in classical micro, and he plays a bit with comparative statics.

If we believe that the prices π_i are given, and S to be a good measure of your total time available in money (at wage *w*) and your other income, the model presents no difference with the textbook micro model. Changes in prices for commodities and the wealth S can be represented in the standard graphs, as follows[12]:



Now, changes in the constraint can be due to changes in the prices for market goods and to changes in the price of time, which in Becker is measured by the wage as the opportunity cost of non work time.

A change in the price vector is depicted in the graph, and makes no difference with traditional micro interpretations. The budget set changes of shape and the changes in prices brings a change in the tangency conditions and hence, changes in the equilibrium.

It is out of any doubt the enormous and fantastic contribution by Becker in 1965, and we just suggest a more general framework. As commented in the literature, the commodity prices defined by Becker present several problems, which are avoided in our model, which is aligned to Pollak´s criticism. Our proposed model is not only more general than Becker, but also it is compatible with Pollak criticisms, which we consider a fantastic contribution.

[12] This graph is copied directly from Becker (1965).

5. DYNAMIC MODEL

After having generalized Becker model, we just construct a *possible* way of making a dynamic model based on it.

Such model is now including stocks of both time and goods for each and one type and possible use of them, and each and their derivatives with respect to time. Such derivatives are, actually the concrete demands for time and goods for each instant of time, and some of the control variables of the problem. As well, is including the production of the time and goods, i.e. the commodities, so then the *happiness* is including the *experience* in using time and goods (modelled as the stock variables), the hedonism of enjoying just the *material* things (the demand for market goods), the stoicism of enjoying *immaterial* things (modelled by time use), and indeed, the fact of enjoying what is produced by combining both material and immaterial things (modelled by commodities, which are produced using time and goods, as in Becker) and the experience on this last too.

Hence, we propose the following model, a 2 step dynamic optimization process:

Let $\mathrm{S}_{X_{n\times q}}, \mathrm{S}_{\mathfrak{I}_{p\times r}}$ be the matrices whose elements $s_{x_{nq}}, s_{T_{pr}}$ are the stock of each one of the respective elements in $X_{n\times q}, \mathfrak{I}_{p\times r}$, i.e., the stock of market goods and time inputs accumulated by the individuals along time, which actually is the individuals´ past experiences.

First step

$$\min_{X_{n\times q},\mathfrak{I}_{p\times r}} E\left(t, S_{X_{n\times q}}, S_{\mathfrak{I}_{p\times r}}, X_{n\times q}, \mathfrak{I}_{p\times r}\right) = \int_{t_0}^{t_f} \left(\sum_{q=1}^{q} \vec{p}^T \vec{x}_q + \sum_{r=1}^{r} \vec{w}^T \vec{T}_r \right) dt$$

$$\dot{S}_{X_{n\times q}} = X_{n\times q}$$

$$\dot{S}_{\mathfrak{I}_{p\times r}} = \mathfrak{I}_{p\times r} \tag{E1}$$

$$s.t.\ f_i\left(\mathrm{S}_{X_{n\times q}}, \mathrm{S}_{\mathfrak{I}_{p\times r}}, X_{n\times q}, \mathfrak{I}_{p\times r}\right) \geq Z_i, i = 1...m$$

which yields:

$$X^*_{n\times q} = \eta\left(t, \vec{p}, \vec{w}, \vec{Z}\right), \mathrm{S}^*_{X_{n\times q}} = \theta\left(t, \vec{p}, \vec{w}, \vec{Z}\right) \tag{a}$$

$$\mathfrak{I}^*_{p\times r} = \phi\left(t, \vec{p}, \vec{w}, \vec{Z}\right), \mathrm{S}^*_{\mathfrak{I}_{p\times r}} = \tau\left(t, \vec{p}, \vec{w}, \vec{Z}\right)$$

and the corresponding Value Function:

$$E^*\left(t, S^*_{X_{n\times q}}, S^*_{\mathfrak{I}_{p\times r}}, X^*_{n\times q}, \mathfrak{I}^*_{p\times r}\right) = E^*\left(t, \vec{p}, \vec{w}, \vec{Z}\right) \tag{b}$$

where $\mathrm{S}_{X_{n\times q}}, \mathrm{S}_{\mathfrak{I}_{p\times r}}$ are the matrices including the stock of different types of time and market goods used in a certain manner

market goods used

Second step

$$\max_{S_{\vec{Z}},\vec{Z}} \int_{t_0}^{t_f} U\left(t, S_{\vec{Z}}, \vec{Z}, \mathrm{S}_{X_{n\times q}}, \mathrm{S}_{\Im_{p\times r}}, X_{n\times q}, \Im_{p\times r}\right)dt$$

$$\overset{\bullet}{S}_{\vec{Z}} = \vec{Z} \qquad \text{(U1)}$$

$$\text{s.t.} \qquad E^*\left(t, \vec{p}, \vec{w}, \vec{Z}\right) \leq \vec{w}^T(t)\cdot \mathrm{S}_{\vec{T}} + \int_{t_0}^{t_f} V dt = L \equiv \text{Full Lifetime Income}$$

which yields the optimal solutions for the second step:

$$\vec{Z}^* = z\left(t, \vec{p}, \vec{w}, \mathrm{S}_{\vec{T}}, V\right),\ S^*_{\vec{Z}} = \varsigma\left(t, \vec{p}, \vec{w}, \mathrm{S}_{\vec{T}}, V\right) \qquad \text{(c)}$$

Inserting $\vec{Z}^*$ in the first step optimal paths and solutions yields:

$$X^*_{n\times q} = \eta\left(t, \vec{p}, \vec{w}, \mathrm{S}_{\vec{T}}, V\right),\ \mathrm{S}^*_{X_{n\times q}} = \theta\left(t, \vec{p}, \vec{w}, \mathrm{S}_{\vec{T}}, V\right) \qquad \text{(d)}$$
$$\Im^*_{p\times r} = \phi\left(t, \vec{p}, \vec{w}, \mathrm{S}_{\vec{T}}, V\right),\ \mathrm{S}^*_{\Im_{p\times r}} = \tau\left(t, \vec{p}, \vec{w}, \mathrm{S}_{\vec{T}}, V\right)$$

Equivalently, in order to get the optimal conditions, the problem can be expressed not using matrices. The next formulation is totally equivalent, and we use it to get and interpret optimal conditions. We do not insist again in expressions in (e) above, which indeed have to hold in this equivalent formulation we present now. For presenting it, it should be noticed that all the elements in all the matrices are receiving a different subscript, in order to order them into the real line, so then they are not meaning any position in any matrix. As well, the vectors of prices and wages are iterated as many times as rows has the market goods and time inputs matrices, in order to assign a concrete price for any type of good (and time) used in a different manner.

Once we pointed this, we just proceed presenting and solving the problem:

First step

$$\max_{\bullet} - E(t,\bullet) = \int_{t_0}^{t_f} -\left(\sum_{k=1}^{n\cdot q} p_k x_k + \sum_{j=1}^{p\cdot r} w_j T_j\right)dt$$
$$\overset{\bullet}{s}_{x_k} = x_k, k = 1,\ldots,n\cdot q$$
$$\overset{\bullet}{s}_{T_j} = T_j,\ j = 1,\ldots,p\cdot r \qquad \text{(E2)}$$
$$s.t.\ \ f_i(\bullet) \geq Z_i, i = 1\ldots m$$

$$H\left(t,\bullet,\vec{\lambda},\vec{\mu}\right) = -\sum_{k=1}^{n\cdot q} p_k x_k - \sum_{j=1}^{p\cdot r} w_j T_j + \sum_{k=1}^{n\cdot q} \lambda_k x_k + \sum_{j=1}^{p\cdot r} \mu_j T_j$$

$$L\left(t,\bullet,\vec{\lambda},\vec{\mu},\vec{\delta}\right) = H\left(\bullet,\vec{\lambda},\vec{\mu}\right) + \sum_{i=1}^{m} \delta_i \left(f_i\left(\bullet\right) - Z_i \right)$$

where $\bullet$ denotes the dependency of each function

with respect to $\left(s_{x_1},...,s_{x_{n\cdot q}},s_{T_1},...,s_{T_{p\cdot r}},x_1,...,x_{n\cdot q},T_1,...,T_{p\cdot r}\right)$

Maximizing the Lagrangian-Hamiltonian $L\left(t,\bullet,\vec{\lambda},\vec{\mu},\vec{\delta}\right)$

we get the following conditions:

$$\frac{\partial L}{\partial x_k} = 0 \Leftrightarrow p_k + \lambda_k = \sum_{i=1}^{m} \delta_i \frac{\partial f_i\left(\bullet\right)}{\partial x_k}, \forall k = 1,...,n\cdot q \qquad \text{(f)}$$

$$\frac{\partial L}{\partial T_j} = 0 \Leftrightarrow w_j + \mu_j = \sum_{i=1}^{m} \delta_i \frac{\partial f_i\left(\bullet\right)}{\partial T_j}, \forall j = 1,..., p\cdot r \qquad \text{(g)}$$

$$-\frac{\partial L}{\partial s_{x_k}} = \dot{\lambda}_k \Leftrightarrow \dot{\lambda}_k = -\sum_{i=1}^{m} \delta_i \frac{\partial f_i\left(\bullet\right)}{\partial s_{x_k}}, \forall k = 1,...,n\cdot q \qquad \text{(h)}$$

$$-\frac{\partial L}{\partial s_{T_j}} = \dot{\mu}_j \Leftrightarrow \dot{\mu}_j = -\sum_{i=1}^{m} \delta_i \frac{\partial f_i\left(\bullet\right)}{\partial s_{T_j}}, \forall j = 1,..., p\cdot r \qquad \text{(i)}$$

Conditions (f) and (g) are indicating that the marginal expenditure of using one more unit of market good (equivalently, time input) has to be equal to the shadow cost of increasing household production marginally due to such an unitary increase in the market good (or time input).

Conditions (h) and (i) are pointing that marginal cost of increasing the household production when increasing the stock of a market good (equivalently, time input) has to be equal to the opposite of the speed of change in the shadow cost of that particular market good (equivalently, time input).

Second step

$$\max_{\vec{Z},\bullet} \int_{t_0}^{t_f} U\left(t, S_{\vec{Z}}, \vec{Z}, \bullet\right) dt$$

$$\dot{S}_{Z_i} = Z_i, i = 1,...,m \qquad \text{(U2)}$$

s.t. $E^*\left(t,\vec{p},\vec{w},\vec{Z}\right) \le \vec{w}^T\left(t\right)\cdot \mathrm{S}_{\vec{T}} + \int_{t_0}^{t_f} V dt = L \equiv$ Full Lifetime Income

$$H\left(t, S_{\vec{Z}}, \vec{Z}, \bullet, \vec{\kappa}\right) = U\left(t, S_{\vec{Z}}, \vec{Z}, \bullet\right) + \sum_{i=1}^{m} \kappa_i Z_i$$

$$L\left(t,S_{\vec{Z}},\vec{Z},\bullet,\vec{\kappa},\pi\right)=H\left(t,S_{\vec{Z}},\vec{Z},\bullet,\vec{\kappa}\right)+\pi\cdot\left(E^*\left(t,\vec{p},\vec{w},\vec{Z}\right)-\vec{w}^T(t)\cdot \mathrm{S}_{\vec{T}}-\int_{t_0}^{t_f}Vdt\right)$$

where $\bullet$ denotes the dependency of each function with respect to $\left(s_{x_1},...,s_{x_{n\cdot q}},s_{T_1},...,s_{T_{p\cdot r}},x_1,...,x_{n\cdot q},T_1,...,T_{p\cdot r}\right)$

Maximizing the Lagrangian-Hamiltonian:

$$L\left(t,S_{\vec{Z}},\vec{Z},\bullet,\vec{\kappa},\pi\right)=U\left(t,S_{\vec{Z}},\vec{Z},\bullet\right)+\sum_{i=1}^{m}\kappa_i Z_i+\pi\cdot\left(E^*\left(t,\vec{p},\vec{w},\vec{Z}\right)-\vec{w}^T(t)\cdot \mathrm{S}_{\vec{T}}-\int_{t_0}^{t_f}Vdt\right)$$

we get the following conditions:

$$(1)\frac{\partial L}{\partial Z_i}=0\Leftrightarrow$$

$$MU_{Z_i}+\kappa_i=-\pi\frac{\partial E^*\left(t,\vec{p},\vec{w},\vec{Z}\right)}{\partial Z_i}\Leftrightarrow-\kappa_i=MU_{Z_i}+\pi\frac{\partial E^*\left(t,\vec{p},\vec{w},\vec{Z}\right)}{\partial Z_i}\equiv\left(\text{commodity rent}\right)\qquad \text{(j)}$$

$,\forall i=1,...,m,$

where $MU_{Z_i}=U'_{Z_i}+\sum_{k=1}^{nq}U'_{x_k}\cdot\frac{\partial x_k}{\partial Z_i}+\sum_{k=1}^{nq}U'_{s_{x_k}}\cdot\frac{\partial s_{x_k}}{\partial Z_i}+\sum_{r=1}^{pr}U'_{T_r}\cdot\frac{\partial T_r}{\partial Z_i}+\sum_{r=1}^{pr}U'_{s_{T_r}}\cdot\frac{\partial s_{T_r}}{\partial Z_i}$

$$(2)\frac{\partial L}{\partial S_{Z_i}}=-\dot{\kappa}_i\Leftrightarrow-\dot{\kappa}_i=U'_{S_{Z_i}}\qquad \text{(k)}$$

differenciating with respect time t in (j), and using (k) we get:

$$\dot{\left(\text{commodity rent}\right)}=U'_{S_{Z_i}}\qquad \text{(ñ)}$$

where the meaning of *commodity rent* is the net satisfaction in terms of utility that individuals experience when a marginal increase of commodities takes place <u>at each instant of time</u>.

The intuition behind conditions (j) is of the type "marginal profit (in terms of utility) equal zero", while the conditions (k) are of the type "marginal utility of increasing the stock of commodities equals to the speed of change in the shadow cost of the commodities.

The intuition behind conditions (ñ) is the following, which we express in terms of a proposition:

PROPOSITION 3 - *(Change of tastes)*: If the individuals enjoy (in contrast, suffer) in terms of utility an increase in the *experience accumulated with certain* [=”*stock of*”] commodities, then the *commodity rent* that each increase in commodity in concrete is yielding at any period of time is growing (diminishing) as time goes by.

This proposition clearly states that the marginal utility of commodities is different as times goes by, under normal and common assumptions for the utility function, and this is so due to the past experiences in having consumed/enjoyed such commodities in the past, which, as a matter of fact, and as a by product, is shaping the endogenous change of tastes. That is so since the same unit increase is not yielding the same net satisfaction at any instant of time, and we conclude that there has been, then, a change of tastes, without having changed the preferences.

Hence, optimal paths along time of the matrix of types and uses of market goods demands and their stock along time, optimal paths along time of the matrix of types and uses of time along time, and optimal vector of commodities and stock of commodities along time are determined over the life cycle. All these paths are functions of economic concepts, as prices, wages and other income, but also functions of social aspects, as the stock of total time available. That distribution of our same and common given amount of time per period among the different types of time that can be set is different across individuals. One might easily think in these differences to be due to gender issues, but traditions and different cultures might be determinant on this aspect, since they might be restricting certain types of time.

In any case, reasonable conclusions about the formation of habits, in the literal sense, are reached. The change on these habits is included into a general framework, and as they are allowed to change endogenously in the model, historical changes can be explained by socio economical factors.

It must be pointed the very philosophical question of the utility function being the same during the whole lifetime. Our claim in here is the following: we believe that, deeply inside, in the end, the inner way of assessing facts is constant. That does not means that we change in other aspects our perspectives, but that is done at a different level, as it is deciding:

- how to get commodities (which in the model is the production process, but decided on the second step, and whose decision is affected by past experiences in the use of each commodity),
- what products to choose (which depends as well upon the past experience) and,
- which is the amount of time we want to spend doing something in a certain manner (which again depends and changes endogenously as a function of our past experiences).

For simplicity we have not included in this paper something very interesting in the laws of motion of the dynamic problem, in its second step. Such a thing is a constant term, varying along time. that one might be interpreted as a psychological impulse, perhaps induced by marketing issues, and/or simply tradition, which individuals cannot control, and in the end of the day is making them to accumulate more (or less) market goods than they would had planned, and as well making them to spend more (or less) time than they planned in doing something in a certain manner. We leave such interesting implication for future research, hopefully within a doctoral dissertation.

6. GENERAL CONCLUSSIONS

In this paper has been generalized Becker's theory of allocation of time, -avoiding criticisms in the sense of Pollak too-, by allowing joint production, something not included in Becker, and pointed by Pollak as the main problem in the household analysis.

After doing that, we just proposed a more general framework, following Pollak´s essence, in which when the individuals are producers they adopt a cost minimization attitude, and when they are consumers, a utility maximizer one. By analyzing the value functions in both steps, particularly the last one (which includes the one obtained in the first step), we end up with two propositions: one stipulating a law of wage compensation, which is based in how individuals value their time, and the other one stating the fact of the existence of some type of time used in a certain manner which is a Giffen good. For the last, several premises have to be fulfilled.

To end, we have just suggested a *possible* way of making a dynamic model of the generalized Becker's theory of allocation of time we propose in the static case. Such a dynamization leads us to very interesting conclusions about patterns for goods and time uses, and of course, patterns for commodities, along the life cycle, which fits in essence with the meaning of *habits*. Optimal conditions are derived and interpreted and their interpretation leads to the existence of changes of tastes.

We provide as well some very interesting insights for future research, which we postpone for a, perhaps, doctoral dissertation.

Raúl G. Sanchis,
Oslo, 09/October/2007

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